

Subject/Course: Math	Grade: 3					
	Suggested Timeline: 6 weeks					
Unit Title: Multiplication and Division with Factors of 2,3,4,5, and 10		This module builds upon the foundation of multiplicative thinking with units started in grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products and representing and solving problems involving multiplication and division involving factors of 2, 3, 4, 5, and 10. The restricted set of facts keeps learning manageable, and also provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, money, length, capacities, and time in the second module.				
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice

Interpret and/or describe products of whole numbers fluently within 100 (up to and including 10 X 10)	Place Value and Properties of Operations	Demonstrate an understanding of the relationship between multiplication and division.	Division, Estimate, Multiplication, Associative Property of Addition, Quotient, Bar Model, Base Ten Numeral Form, Base ten numerals, Column, Commutative Property of Addition, Compatible numbers, Difference, Digit, Equation, Estimate, Even number, Expanded form, Hundreds, Factor, Multiple, Inverse operations, Number line, Odd number, Ones, Order of operations, Parentheses, Place value, Reasonableness, Regroup, Round a whole number, Row, Standard form, Subtract, Tens, Variable, Even number, Fact family, Multilicative identity property of 1, Multiply, Odd number, Product, Related facts, Sum, Zero property of multiplication, Pattern, Unknown, Square number, Array, Infinite		Represent and solve problems involving multiplication and division.	MP #1,2,3,4,5,6,7,8
Interpret and/or describe whole-number quotients of whole numbers (limit dividends through 50, and limit divisors and quotients through 10).		Demonstrate fluency.			Understand properties of multiplication and the relationship between multiplication and division.	

Apply the commutative property of multiplication (not identification or definition of the property)		Represent and solve problems.			Demonstrate multiplication and division fluency.	
Apply the associative property of multiplication (not identification or definition of the property)		Identify and explain patterns in arithmetic (including addition and subtraction).				
Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers						
Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers						
Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers						
Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols)						

<p>Identify arithmetic patterns (including patterns in the addition table or multiplication table) and/or explain those using properties of operations</p>						
<p>Create or match a story to a given combination of symbols and numbers</p>						
<p>Identify the missing symbol that makes a number sentence true</p>						
<p>Use multiplication (up to and including 10×10) and/or division (limit dividends through 50, and limit divisors and quotients through 10) to solve word problems in situations involving equal groups, arrays, and/or measurement quantities</p>						
<p>Determine the unknown whole number in a multiplication (up to and including 10×10) or division (limit dividends through 50), and limit divisors and quotients through 10) equation relating three whole numbers</p>						
<p>Interpret and/or model division as a multiplication equation with an unknown factor</p>						

Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Apply place value understanding and properties of operations to perform multi-digit arithmetic.	Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as $15 \div 3 = ?$ or $15 = x \cdot 3$.	The use of models is essential in helping students eliminate this understanding.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Students also think that $3 \div 15 = 5$ and $15 \div 3 = 5$ are the same equations.	Presenting students with multiple situations in which they select the symbol and explain its meaning.
	The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation.	What the symbol represents will counter this misconception.

Subject/Course: Math	Grade: 2					
	Suggested Timeline: 6 weeks					
Unit Title: Problem solving with Mass, Time, Capacity, Length, and Money		<p>Module 2, which focuses on measurement, again provides students with internalization time for learning the 2, 3, 4, 5, and 10 facts as part of their fluency activities. Students can also take this time to work with place value, comparison and rounding concepts. The goal is to develop students' number sense well enough that they can build proportional bar diagrams used in solving word problems in Grade 3 and beyond (e.g., "If this bar represents 62 kg, then a bar representing 35 kg needs to be slightly longer than half the 62 kg bar..."). Drawing the relative sizes of the lengths of two bars also prepares students to locate fractions on a number line in Module 5 (where they learn to locate the points $\frac{1}{3}$ and $\frac{1}{5}$ on the number line relative to each other and relative to the whole unit).</p>				
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
<p>Make change for an amount up to \$5.00 with no more than \$2.00 change given (penny, nickel, dime, quarter, and dollar)</p>	<p>Measurement</p>	<p>Make estimations.</p>	<p>Estimate, Liquid volume, Mass, Length, Temperature, Celsius, Fahrenheit, Degree, Interval, Change, Value, Elapsed time, Second, Minute, Hour, Coin values, Round, Benchmark, Time, Weight, Capacity, Inches, Feet, Yard, Cups, Pints, Quarts, Gallons, Ounces, Pounds, Metric units, Grams, Kilograms, Appropriate unit of measure, Estimating measures, Exact . estimate</p>		<p>Tell and write time to the nearest minute and solve problems by calculating time intervals.</p>	<p>MP # 1,2,4,5,6,7,8</p>

Round amounts of money to the nearest dollar	Time	Distinguish between linear and area measurements.			Solve problems and make change involving money using a combination of coins and bills.	
Tell, show and/or write time (analog) to the nearest minute	Money (Coins and Bills)	Solve problems.				
Calculate elapsed time to the minute in a given situation (total elapsed time limited to 60 minutes or less)		Tell and write time to nearest minute.				
Measure and estimate liquid volumes and masses of objects using standard units (cups, pints, quarts, gallons, ounces, and pounds and metric units, grams, and kilograms)		Calculate time intervals.				
Compare total values of combinations of coins (penny, nickel, dime, quarter) and/or dollar bills less than \$5.00		Make change using combination of coins and bills.			Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.	
Add, subtract, multiply, and divide to solve one-step word problems involving masses or liquid volumes that are given in the same units						
Use a ruler to measure lengths to the nearest quarter inch or centimeter						

Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Represent and solve problems involving multiplication and division.	Avoid the use of paper plate clocks. This is not adequately represented on student made clocks.	Students need to see the actual relationship between the hour and the minute hand. Model explicitly.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Students forget to label the measurement or choose the incorrect unit.	Students need to have visual hooks of benchmarks established from experiences, which build conceptual understanding. Example: Taping a square yard on the floor, acting out a square foot using arms or yarn. Putting cubic units into a rectangular prism.
	Students often focus on size to determine estimates of mass. They can be confused by a big fluffy object and a tiny dense object.	Because students cannot tell actual mass until they have handled an object, it is important that teachers do not ask students to estimate the mass of objects until they have had the opportunity to lift the objects and then make an estimate of the mass.

	Students confuse whether to use metric or standard units and do not line up the ruler at the appropriate mark.	After modeling explicitly with the group, follow up with small group formative assessment, and then individual assessment of using a ruler correctly. First, have students estimate, then measure accurately to check for proper alignment, etc.
	Students have difficulty making reasonable estimations.	Students need to have practice unitizing large areas. (They need to have established reference or benchmark for a mg (grain of sand weight), an inch (a fingertip), etc.
Apply place value understanding and properties of operations to perform multi-digit arithmetic	Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams maybe read as 81 grams. Students realize it is one away from 80, but do not think of it as 79grams.	Students need to practice skip counting by a variety of intervals. They need to make sense of how the scale needs to be interpreted based on the labeled numbers on the scale.

Subject/Course: Math	Grade: 3					
	Suggested Timeline: weeks					
Unit Title: Multiplication and Division with Factors of 6,7,8, and 9		<p>Students learn the remaining multiplication and division facts in Module 3 as they continue to develop their understanding of multiplication and division strategies within 100 and use those strategies to solve two-step word problems. The “2, 3, 4, 5 and 10 facts” module (Module 1) and the “6, 7, 8 and 9 facts” module (Module 3) both provide important, sustained time for work in understanding the structure of rectangular arrays to prepare students for area in Module 4. This work is necessary because students initially find it difficult to distinguish the different squares in a rectangular array area model (the third array in the picture below), count them and recognize that the count is related to multiplication. Modules 1 and 3 slowly build up to a rectangular array area model using hands-on rectangular arrays (i.e., a Rekenrek) and/or pictures of rectangular arrays involving objects only (stars, disks, etc.)—all in the context of learning multiplication and division.</p>				
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
Interpret and/or describe products of whole numbers fluently within 100 (up to and including 10 X 10)	Multiplication and Division	Demonstrate an understanding of properties of multiplication.	Division, Estimate, Multiplication, Factor, Product, Pattern, Multiple, Unknown, Quotient, Commutative, Associative, Area model, Equation, Number model, Number sequence		Represent and solve problems involving multiplication and division.	MP # 1,2,3,4,5,6,7,8
Interpret and/or describe whole-number quotients of whole numbers (limit dividends through 50, and limit divisors and quotients through 10)		Demonstrate an understanding of the relationship between multiplication and division.			Understand properties of multiplication and the relationship between multiplication and division.	
Apply the commutative property of multiplication (not identification or definition of the property)		Demonstrate fluency.			Demonstrate multiplication and division fluency.	

<p>Apply the associative property of multiplication (not identification or definition of the property)</p>						
<p>Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.</p>						
<p>Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.</p>						
<p>Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.</p>						
<p>Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).</p>						
<p>Identify arithmetic patterns (including patterns in the addition table or multiplication table) and/or explain them using properties of operations.</p>						

<p>Create or match a story to a given combination of symbols and numbers.</p>						
<p>Identify the missing symbol that makes a number sentence true.</p>						
<p>Use multiplication (up to and including 10×10) and/or division (limit dividends through 50, and limit divisors and quotients through 10) to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.</p>						
<p>Determine the unknown whole number in a multiplication (up to and including 10×10) or division (limit dividends through 50), and limit divisors and quotients through 10) equation relating three whole numbers.</p>						
<p>Interpret and/or model division as a multiplication equation with an unknown factor</p>						

Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Apply place value understanding and properties of operations to perform multi-digit arithmetic.	Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as $18/3 = ?$ or $18 = ? \times 3$.	The use of models is essential in helping students eliminate this understanding.
Understand properties of multiplication and the relationship between multiplication and division.	Students also think that $3 \div 18 = 6$ and $18 \div 3 = 6$ are the same equations.	Presenting students with multiple situations in which they select the symbol and explain the meaning.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation.	What the symbol represents will counter this misconception.
	Multiplication and Division do not BOTH follow the commutative and associative property. Some students may both properties apply to both operations.	Examples of the commutative and associative properties for multiplication must be explored, and then use inquiry to have students determine if this rule will apply to division.

Subject/Course: Math	Grade: 3					
	Suggested Timeline: 5 weeks					
Unit Title: Multiplication and Area		By Module 4, students are ready to investigate area and the formula for the area of a rectangle. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (as in Module 3).				
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
Measure areas by counting unit squares (square centimeter, square meter, square inch, square foot, and non-standards units)	Place Value and Properties of Operations	Demonstrate an understanding of properties of multiplication.	Factor, Product, Multiple, Rectangular array, Square unit, Length, Width, Rows, Columns, Commutative (in relation to $L \times W = W \times L$), Dimension, Square numbers, Equation, Patterns, Repeated addition, Fact family, Area, Perimeter		Determine the area of a rectangle and apply the concept to multiplication and to addition.	MP # 1,2,3,4,5,6,7,8
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning	Patterns	Represent and solve problems.				
		Identify and explain patterns in arithmetic (including addition and subtraction).				

		Determine the area of a rectangle as it relates to multiplication and addition.				
		Distinguish between linear and area measurements.				

Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Represent and solve problems involving multiplication and division.	Students confuse area and perimeter.	Students must be explicitly taught that perimeter is the “edge” of a color tile when building a rectangular array, while area is represented by the surface that covers a color tile. Students count the edges around a shape to find the perimeter in linear units, while they count the squares to find the number of square units that cover a shape.
Understand properties of multiplication and the relationship between multiplication and division.	Students choose the wrong operation to calculate area or perimeter.	After creating a rectangle with a rubber band on a geoboard, students can count the distance from “peg to peg” to find perimeter of the shape, while counting the squares (4 peg square sections) to find the area.

<p>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p>	<p>Students do not label square units and units correctly.</p>	<p>Once students understand conceptually the difference between area and perimeter, gradually move them to discovering patterns/short cuts to develop the formula for area of a rectangle. Follow up all activities with a word wall card, labeled with a visual of the perimeter or area activity in which the students were engaged.</p>
		<p>Discussion must clearly identify when to multiply and why and when to add and why. Probe students thinking with questions.</p>
		<p>Much practice must be done with manipulatives in order for students to understand that “flat squares” that cover a shape need to be labeled in “square units” and distance around the shape are the total of the edges or “units.”</p>

Subject/Course: Math	Grade: 3					
	Suggested Timeline: 6 weeks					

Unit Title:
Fractions as Numbers on the Number Line

The goal of Module 5 is for students to transition from thinking of fractions as parts of a figure to points on a number line. To make that jump, students think of fractions as being constructed out of unit fractions: “1 fourth” is the length of a segment on the number line such that the length of four concatenated fourth segments on the line equals 1 (the whole). Once the unit “1 fourth” has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, etc. Students also compare fractions, find equivalent fractions in special cases, and solve problems that involve comparing fractions.

I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
Demonstrate that when a whole or set is partitioned into y equal parts, the fraction $1/y$ represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit the denominators to 2,3,4,6,and 8; limit numerators to whole numbers less than the denominator; no simplification necessary)	Fractions	Represent fractions on a number line.	Numerator, Denominator, Division, Equivalent fractions, Fraction bar, Number line and open number, Line, Symmetric, Symmetry, Equal parts, Equal length, Distance, Part, Whole, Increment		Explore and develop an understanding of fractions as numbers.	MP # 1,2,3,4,5,6,7,8
Represent fractions on a number line (limit the denominators to 2,3,4,6, and 8; limit numerators to whole numbers less than the denominator; no simplification necessary)		Represent and generate equivalent fractions.			Understand properties of multiplication and the relationship between multiplication and division.	

<p>Recognize and generate simple equivalent fractions (limit the denominators to 1,2,3,4,6, and 8; limit numerators to whole numbers less than the denominator)</p>		<p>Compare fractions with the same numerator or same denominator.</p>				
<p>Express whole numbers as fractions, and/or generate fractions that are equivalent to whole numbers (limit the denominators to 1,2,3,4,6, and 8; limit numerators to whole numbers less than the denominator)</p>		<p>Partition two-dimensional shapes into equal parts.</p>				
<p>Compare two fractions with the same denominator (limit the denominators to 1,2,3,4,6, and 8), using the symbols $>$, $=$, or</p>		<p>Express the area of a partition as a unit fraction of the whole.</p>				
<p>Round two- and three-digit whole numbers to the nearest ten or hundred, respectively</p>						
<p>Add two- and three-digit whole numbers (limit sums from 100 through 1,000), and/or subtract two- and three-digit numbers from three-digit whole numbers.</p>						
<p>Multiply one-digit whole numbers by two-digit multiples of 10 (from 10 through 90)</p>						

Order a set of whole numbers from least to greatest or greatest to least (up through 9,999; limit sets to no more than four numbers)

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Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Apply place value understanding and properties of operations to perform multi-digit arithmetic.	The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is.	The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.
	Students think all shapes can be divided the same way (three-eighths, and so on).	Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a (non-equilateral) triangle into eighths.
	Students may think that because a shape is divided into 3 parts the parts are thirds. (All parts must be congruent).	Provide oral directions for folding the triangle: Note: This will work for an equilateral triangle. Ask students: Does this work for all types of triangles?

		Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
		Fold in this manner two more times.
		Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, Example/nonexam
		ple strategy to show equal portions of a number line or other shape.

Subject/Course: Math	Grade: 3					
	Suggested Timeline: 5 weeks					
Unit Title: Collecting and Displaying Data		In Module 6, students leave the world of exact measurements behind. By applying their knowledge of fractions from Module 5, they estimate lengths to the nearest halves and fourths of an inch and record that information in bar graphs and line plots. This module also prepares students for the multiplicative comparison problems of grade 4 by asking students “how many more” and “how many less” questions of scaled bar graphs.				
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
Complete a scaled pictograph and a scaled bar graph to represent a data set with several categories (scales limited to 1,2,3 and 10)	Data Displays	Make estimations.	Bar graph, Chart, Difference, Graph, Increment, Interval, Line plot graph, Pictograph, Round, Data, Greatest, Least		Represent and interpret data using tally charts, tables, pictographs, line plots, and bar graphs.	MP # 1,2,3,4,5,6,7,8
Solve one- and two-step problems using information to interpret data presented in scaled pictographs and scaled bar graphs		Represent and interpret data using various displays.				
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Display the data by making a line plot, where the horizontal scale is marked in appropriate units - whole numbers, halves or quarters		Solve problems.				

Translate information from one type of display to another. Limit to pictographs, tally charts, bar graphs, and tables.

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Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.	Although intervals on a bar graph are not in single units, students count each square as one.	To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip- counting when determining the value of a bar since the scale is not in single units.

Subject/Course: Math	Grade: 3 Suggested Timeline: 4 weeks					
Unit Title: Word Problems with Geometry and Measurement						
I Can Statements / Essential Questions / Objectives	Content / Concepts	Skills / Competencies	Vocabulary	Assessments	Focus Standards	Standards for Math Practice
Explain that shapes in different categories may share attributes, and that the shared attributes can define a larger category	Two and Three Dimensional Figures	Compare shapes.	Point, line, ray, line segment, plane, intersecting, parallelm, right angle, solid figure, vertex, symmetry, flip, slide, turn, symmetrical, acute, obtuse, straight, polygon, congruent, perimeter, area, volume, face, edge, prism, pyramid, shpere, cylinder, cube, cone, angle, circle, square, triangle, rectangle, pentagon, hexagon, octagon		Identify, compare, and classify shapes and their attributes.	MP # 1,2,3,4,5,6,7
Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and/or draw examples of quadrilaterals that do not belong to any of these subcategories Partition shapes into parts with equal areas Express the area of each part as a unit fraction of the whole		Identify and classify shapes and their attributes.			Use the understanding of fractions to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.	
					Solve problems involving perimeters of polygons and distinguish between linear and area measures.	

<p>Solve two-step problems using the four operations (expressions are not explicitly states) Limit to problems with whole numbers and having whole-number answers</p>						
<p>Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols)</p>						
<p>Use a ruler to measure lengths to the nearest quarter inch or centimeter</p>						
<p>Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, exhibiting rectangles with the same perimeter and different areas, and exhibiting rectangles with the same area and different perimeters. Use the same units throughout the problem</p>						

Important Standards Addressed in This Unit	Misconceptions	Proper Conceptions
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter.	Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.
Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.	Students may identify a square as a “non-rectangle” or a “non-rhombus” based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well.	Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

